influenced by the body becomes larger than the actual one. In order to correct for this a shock shape must be introduced which closely approximates the actual shock. In the example of Fig. 2, at a Mach number of 2.5, a conical shock shape was used. The agreement between experimental and theoretical values of interference lift is good (Fig. 2).

The feature of the negative pressure coefficient ahead of the maximum cross section of the body, and the resulting negative interference lift region previously described is exhibited by most of the parabolic and power-law bodies of revolution (minimum drag bodies) used in supersonic flow. Therefore, it should be considered in the analysis and optimization of the forementioned interference configuration.

There is no effect of the wing upon the body for the spacing just considered. For closer spacing there is a negative lift acting on the body; however, the lift on the wing increases, and the total lift is still positive.

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Magnetic Induction Parameter for Lorentz Accelerators

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The thrust per unit cross section area p_f developed by Lorentz accelerators is in the first approximation equal to the cross product of the magnetic induction B and the linear electric current density j measured in the transverse plane of the flow:

$$p_f = j \times B = \dot{m}(v_2 - v_1) \tag{1}$$

It is also equal to the change in momentum of the working fluid with a mass flow density of $\dot{m}$ and with entrance and exit velocities of v_1 and v_2, respectively. Equation (1) is valid for the range in which secondary effects are negligible, i.e., as long as the magnetic induction B is small. The optimum amount of B which produces maximum thrust output with a given electric current density depends, therefore, on secondary effects. Inclusion of phenomena such as ion slip and Hall currents in the derivation of Eq. (1) leads to a complicated analysis.[1]

A different approach to this problem can be based on the hypothesis that secondary effects become significant when high efficiencies in the energy conversion process are attempted. The optimum amount for B can be determined directly from the change of available electric energy into kinetic energy of the working fluid. The electric power input per unit cross section area of the accelerator q has to be equal to or greater than the increase in kinetic energy of the working fluid plus the power required to increase its degree of ionization by the fraction $\Delta\alpha$:

$$q \geq \epsilon\dot{m}\Delta\alpha + 0.5\dot{m}(v_2^2 - v_1^2) \tag{2}$$

with ϵ being the ionization (and dissociation) energy of the working fluid. Equations (1) and (2) are combined to eliminate v_2:

$$(j \times B)/\dot{m} \leq \{2\epsilon[(q/\dot{m}\epsilon) - \Delta\alpha] + v_1^2\}^{1/2} - v_1 \tag{3}$$

Equation (3) can be used to determine the proper amount of the magnetic induction B for the experiment. Demetriades[2] and Ziemer determined the optimum amount of B experimentally for the specific conditions of their test stand and obtained $B = 1840$ gauss. Equation (3) yields for the same test conditions $B = 1780$ gauss, if a (certainly too large) $\Delta\alpha = 1$ is assumed arbitrarily. The experimentally determined amount of $B = 1840$ gauss, when introduced into Eq. (3), results for this specific test condition in a $\Delta\alpha = 0.75$ instead of the here-assumed $\Delta\alpha = 1$.

In the event that cross-field accelerators with high thrust output are under consideration, Eq. (3) can be simplified by neglecting v_1 and $\Delta\alpha$ against the other terms, and a magnetic performance parameter σ can be established:

$$\sigma = B(j/E\dot{m})^{1/2} = Bh(I/VM)^{1/2} \leq 2^{1/2} \tag{4}$$

with E being the voltage potential. In the second term, the distance h, the voltage drop V, and the total current I are measured between the electrodes of the accelerator, and M designates the total mass flow rate through the unit. This parameter σ may turn out to be useful not only in comparing performance characteristics of different $j \times B$ accelerators but also in the adjustment and calibration of local nonuniformities in the properties of the same unit.

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Received March 6, 1963.

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Earth Albedo Input to Flat Plates

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In this short note, the results of an analysis considering the earth reflected solar radiation incident upon a spinning flat plate are presented briefly. A general description of the problem is given, as well as a definition of all of the geometrical parameters, even though the final result itself is not given explicitly. However, the final integral expression is given, as well as the expressions for determining the integration limits. In actual practice, it proves to be rather easy to perform the integration with the aid of a computer. The parameters introduced succeed in defining the orientation of the surface with respect to the earth. No attempt is made here to give these parameters in terms of orbital parameters. Even so, unfortunately, it would not relieve the reader from the troublesome task of determining the remainder of the parameters from analysis of such data as time of launch, point of launch, injection angle, etc., for any particular problem that he may wish to consider.

Nomenclature

S = mean solar constant
$\mathbf{s}$ = solar vector

Received March 8, 1963.

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α = mean albedo of earth
$\mathbf{r}$ = vector between plate and earth's center
$\mathbf{A}$ = normal to plate
ϑ_s = angle between $\mathbf{s}$ and $\mathbf{r}$
λ = angle between $\mathbf{A}$ and $\mathbf{r}$
φ = azimuthal coordinate of $d\mathbf{\Sigma}$
ξ = angle between $d\mathbf{\Sigma}$ and $-\boldsymbol{\varrho}$
$d\mathbf{\Sigma}$ = element of terrestrial surface area
$\boldsymbol{\varrho}$ = vector between plate and $d\mathbf{\Sigma}$
β = angle between $-\mathbf{s}$ and $d\mathbf{\Sigma}$
ϑ = colatitudinal coordinate of $d\mathbf{\Sigma}$
η = angle between $\mathbf{A}$ and $\boldsymbol{\varrho}$
r_0 = radial coordinate of $d\mathbf{\Sigma}$, $\equiv 1$
σ = angle between $\mathbf{r}$ and $\boldsymbol{\varrho}$

Introduction

IN a recent paper,[1] the problem of determining the solar radiation reflected by the earth which is incident upon a spherical satellite has been discussed. In this note, the author proposes to consider (on the basis of the model used before, i.e., that the earth is a uniform, diffuse reflector) the energy input to an arbitrarily oriented flat plate. The problem is essentially one of geometry, and the author defines and gives expressions for all those parameters necessary for the description of the problem, as well as the general expression that must be integrated. Apart from the obvious application of the methods to determine the total power input to a satellite component such as a solar cell array, the results of the work as presented herein lend themselves to determining the energy distribution upon any of the surfaces of a satellite or space vehicle for which the orientational parameters can be given. Since these vehicles, in general, are spinning, the incident energy determination, to be useful, must be averaged over a spin period. This precludes treatment of surfaces that undergo shielding by other members of the satellite during a portion of each spin period, unless the values of the spin angle at which the surface is eclipsed can be determined. Since knowledge of the spin shadow parameters makes it possible to treat cases of partial shielding by making only simple modifications in the results, these will not be treated here. However, as a result of the spin, many surfaces do suffer self-shielding (present their backsides, as it were) during a portion of each revolution, and these cases are treated. Incidentally, a satellite surface spinning about an axis passing through its center presents the same physical picture to the earth if it spins about an axis parallel to the first but displaced from it. Therefore, no further mention is made of either case. In this work, the actual solution of the problem is left to an IBM 7090 computer. Since the number of parameters involved is rather large, it is impractical to attempt to present here the results of any sample calculations either graphically or in tabular form.

Analysis

In Fig. 1, the needed parameters are defined, and the geometry is delineated. The associated definitions are given in the nomenclature.

The general expression for the incident reflected radiation to a plate of unit area is

$$P = \int_{\Sigma} \frac{S\alpha \cos\beta}{\pi} \cos\xi \frac{\cos\eta}{\rho^2}\, d\Sigma \tag{1}$$

In Eq. (1), $S\alpha \cos\beta d\Sigma$ gives the amount of incident solar energy reflected by $d\Sigma$. This quantity multiplied by $\cos\xi/\pi$ gives the amount of energy reflected by $d\mathbf{\Sigma}$ in the direction of the plate per unit solid angle. The factor $\cos\eta/\rho^2$ gives the solid angle subtended at $d\mathbf{\Sigma}$ by the plate of unit area. Equation (1) becomes[1, 2]

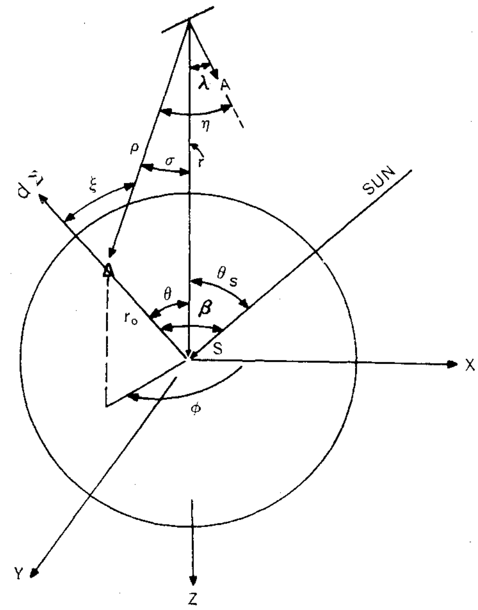

Fig. 1 Earth-satellite geometry for the general case

In Eq. (2), the range of the ϑ integration is $0 \leqslant \vartheta \leqslant \vartheta_m$, where $\vartheta_m = \cos^{-1}(1/r)$, and the range of the ϑ integration is $0 \leqslant \varphi \leqslant \varphi_m$. The determination of φ_m is not quite so simple. Upon some reflection, it can be seen that φ_m is determined by the fact that the side of the plate in question no longer receives any reflected solar radiation from a particular element of surface area $d\mathbf{\Sigma}$, the azimuthal coordinate of which is φ, and that this situation comes about in two ways: 1) the source function $S\alpha \cos \beta d\Sigma$ goes to zero, and 2) the solid angle term $\cos\eta/\rho^2$ goes to zero. The first of these is found by setting $\cos\beta = 0$, from which one has [Ref. 1, Eq. (5)]

$$\varphi_{m1} = \cos^{-1}(-\cot\vartheta \cot\vartheta_s) \tag{3}$$

and the second by equating $\cos\eta$ to zero. However, before this can be done, it becomes necessary to introduce several new parameters and to define the value of $\cos\eta$ in terms of these.

At this point the following new quantities are introduced:

$\boldsymbol{\omega}$ = satellite spin axis (assumed to coincide with its symmetry axis); it coincides with and has same direction as the angular momentum vector

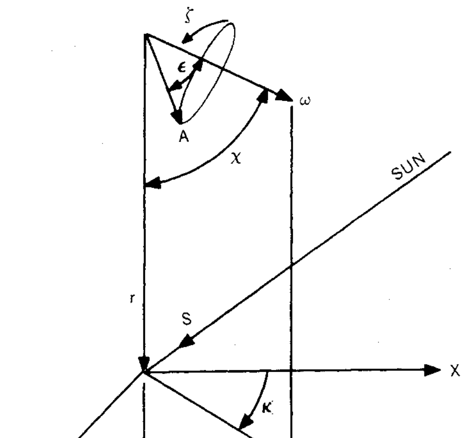

Fig. 2 Definition of the spin parameters

$$P = \frac{S\alpha}{\pi} \int_{\vartheta} \int_{\varphi} \frac{(r \cos\vartheta - 1)(\cos\vartheta \cos\vartheta_s + \sin\vartheta \sin\vartheta_s \cos\varphi)}{(r^2 + 1 - 2r \cos\vartheta)^{3/2}} \cos\eta \sin\vartheta\, d\vartheta d\varphi \tag{2}$$

χ = the angle between $\mathbf{r}$ and $\boldsymbol{\omega}$
ϵ = the angle between $\boldsymbol{\omega}$ and $\mathbf{A}$ (a fixed quantity for most satellites)
ζ = azimuthal angle of spin of $\mathbf{A}$ about $\boldsymbol{\omega}$
κ = the angle of rotation of $\boldsymbol{\omega}$ about $\mathbf{r}$, which gives the orientation of $\boldsymbol{\omega}$ with respect to the $\mathbf{s}$, $\mathbf{r}$ plane (defined similar to φ)

In Fig. 2, these quantities are shown, and a coordinate system (X,Y,Z) is defined, from which φ and κ are defined. As $\mathbf{A}$ rotates about $\boldsymbol{\omega}$, it coincides with the $\boldsymbol{\omega}$, $\mathbf{r}$ plane on two occasions. The zero value of ζ is taken to be the situation for which $\mathbf{A}$ lies in the $\boldsymbol{\omega}$,$\mathbf{r}$ plane, and its motion is in the positive κ direction. Then,

$$\cos\eta = \frac{(r - \cos\vartheta)}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}}(\cos\chi\cos\epsilon - \sin\chi\sin\epsilon\cos\zeta) + \frac{\sin\vartheta}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}}[\cos\varphi\cos\kappa\cos\epsilon\sin\chi + \sin\varphi\sin\kappa\cos\epsilon\sin\chi + \cos\varphi\cos\kappa\sin\epsilon\cos\chi\cos\zeta + \sin\varphi\sin\kappa\sin\epsilon\cos\chi\cos\zeta + \sin\varphi\cos\kappa\sin\epsilon\sin\zeta - \cos\varphi\sin\kappa\sin\epsilon\sin\zeta] \quad (4)$$

Setting the right side of Eq. (4) to zero, one has

$$\cos\varphi_{m2} = -FG \pm H(G^2 + H^2 - F^2)^{1/2}/(G^2 + H^2) \quad (5)$$

where

$$F = [(r - \cos\vartheta)/\sin\vartheta](\cos\chi\cos\epsilon - \sin\chi\sin\epsilon\cos\zeta)$$
$$G = (\cos\kappa\cos\epsilon\sin\chi + \cos\kappa\sin\epsilon\cos\chi\cos\zeta - \sin\kappa\sin\epsilon\sin\zeta)$$
$$H = (\sin\kappa\cos\epsilon\sin\chi + \sin\kappa\sin\epsilon\cos\chi\cos\zeta + \cos\kappa\sin\epsilon\sin\zeta)$$

Since φ_{m1} is determined by the source function that is symmetric about the $\mathbf{s}$,$\mathbf{r}$ plane, the values of φ in the range $0 \leqslant \varphi \leqslant 2\pi$ which contribute to the input are $0 \leqslant \varphi \leqslant \varphi_{m1}$, $(2\pi - \varphi_{m1}) \leqslant \varphi \leqslant 2\pi$, and $\varphi_{m1} \leqslant \pi$. However, the solution of Eq. (5) for φ_{m2} yields two roots symmetric about the $\boldsymbol{\omega}$,r plane. Since the function is multivalued, the problem is to find them. For any given problem these can be determined upon careful examination of the physical picture and with the aid of the following: 1) one value φ_{m2}', say, must lie in the range $\kappa \leqslant \varphi \leqslant \pi + \kappa$, and the other, φ_{m2}'', must lie in the range $\pi + \kappa \leqslant \varphi \leqslant 2\pi + \kappa$; and 2) the values of φ which contribute to the input are $\kappa \leqslant \varphi \leqslant \varphi_{m2}'$ and $(\kappa + 2\pi - \varphi_{m2}') \leqslant \varphi \leqslant 2\pi + \kappa$. In practice, the computer program is written in such a way that it is not necessary actually to determine the limits. The only values of φ which contribute are those lying in the overlapping regions of the two ranges given by φ_{m1} and φ_{m2}. Therefore, for the φ integration, the program that has been used is written in such a way that, for each value of φ (remember that the computer calculates the function for incremental steps of φ and adds them), the computer makes a check of both $\cos\beta$ and $\cos\eta$. If *both* are $\leqslant 1$ and >0, then that value of φ contributes, and the complete computation is made and stored. If one or both is $\leqslant 0$, a zero is entered. In this manner the computer runs through the entire range of φ from 0 to 2π.

For the special case when $\chi = 0$ (i.e., the spin axis coincides with $\mathbf{r}$), the expression to be used for $\cos\eta$ is

$$\cos\eta = \frac{\sin\epsilon\sin\vartheta(\sin\zeta\sin\varphi + \cos\zeta\cos\varphi)}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} + \frac{(r - \cos\vartheta)\cos\epsilon}{(r^2 + 1 - 2r\cos\vartheta)^{1/2}} \quad (6)$$

Equation (6) follows from Eq. (4) by making the substitutions $\chi = 0$ and $\kappa = 0$. The latter is required because, for the case when $\chi = 0$, the angle κ has no meaning and therefore is given here the value of zero.[3] In addition, it might be well to point out here that many combinations of values of the parameters ϵ, χ, κ yield an identical physical picture as another set. Therefore, the only range of values for the parameters that need be considered is $0 \leqslant \chi \leqslant \pi$, $0 \leqslant \epsilon \leqslant \pi/2$, and $0 \leqslant \kappa \leqslant \pi$. For any value of $\epsilon > \pi/2$, the same physical picture is obtained if χ is replaced by $(\pi - \chi)$, ϵ by $(\pi - \epsilon)$, and κ by $(\pi + \kappa)$. For any value of $\kappa > \pi$, one can replace it by $(2\pi - \kappa)$, all other parameters remaining the same. The fact that a value of $\chi > \pi$ is equivalent to $(2\pi - \chi)$ is obvious.

The average over a spin period $\langle P \rangle$ defined by

$$\langle P \rangle = \frac{1}{2\pi}\int_0^{2\pi} P(\zeta)d\zeta$$

is done quite simply by the computer.

Discussion

In the foregoing, the equation for P, even though it has not been solved explicitly, is an exact expression only for the geometric aspects of the problem. The assumption that earth is a spherical, diffuse reflector is necessary if the equation is not to be much more complicated than it is. However, these assumptions would seem to be not as serious as the approximations that the reflectivity is latitude- and longitude-independent and that there is no time-dependency. This undoubtedly is not true. If the ϑ and φ dependence were known, the expression could be modified readily, resulting in essentially no additional labor for the computer integrations. The time variation is, of course, much greater, depending upon such things as cloud cover, cloud location with respect to the sun, etc. However, in assuming a certain amount of spatial uniformity (as is done in the present model), time changes can be handled because the average albedo enters the expressions only as a multiplicative constant, and the result can be changed accordingly.

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Real Gas Performance of Helium Drivers

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FOR lunar and planetary entry problems, a basic research tool in problems involving convective heat transfer and radiative heat transfer is the electrically driven hypervelocity shock tube.[1, 2] The driver contains helium, which is heated by the rapid discharge of electrical energy. Previous analysis of the performance of electrically driven shock tubes has been limited to the perfect gas assumption because of the lack of thermodynamic data on helium. During the course of the

Received March 8, 1963.
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